VOLUME 76 SEPARATE No. 35

PROCEEDINGS

AMERICAN SOCIETY
OF
CIVIL ENGINEERS

SEPTEMBER, 1950

DESIGN CURVES FOR FOOTINGS ON SOIL

By Winfield A. McCracken, Assoc. M. ASCE

SOIL MECHANICS DIVISION

Headquarters of the Society
33 W. 39th St.
New York 18, N.Y.

PRICE $0.50 PER COPY

Published at Prince and Lemon Streets, Lancaster, Pa., by the American Society of Civil Engineers. Editorial and General Offices at 33 West Thirty-ninth Street, New York 18, N. Y. Reprints from this publication may be made on condition that the full title of paper, name of author, page reference, and date of publication by the Society are given.

Proceedings—Vol. 76 September, 1950 Separate No. 35

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

PAPERS

DESIGN CURVES FOR FOOTINGS ON SOIL

By Winfield A. McCracken,[1] Assoc. M. ASCE

Synopsis

Many design tables for footings have been published. However, where such tables are comprehensive, they become cumbersome, and, because of this cumbersomeness, are difficult to use.

A study of the Code of the American Concrete Institute (ACI 318-47) shows that the depth-width ratio and the reinforcing are controlled (*a*) in footings of uniform thickness by bond, and (*b*) in sloped, or stepped footings by bond or shear, or both. It also shows that the depth-width ratio for plain concrete footings is controlled by tension on a section at the face of the column.

The method presented herein is more flexible and less cumbersome than any method involving tables, and it shows the relation between the code requirements clearly. For reinforced footings, only three sets of curves containing seven variables need to be used. For plain concrete footings, only one set of curves with three variables is required.

Following a derivation of equations, the curves are explained and an example of their use is given to make the method of procedure clear. This section is followed by a brief explanation of the adaptation of the curves to rectangular, one-way, and continuous footings.

With the exceptions, as explained under the heading, "Derivation of Equations for Curves," the ACI code is used throughout.

Derivation of Equations for Curves

Notation.—The symbols in this paper, defined where they first appear (by illustration or in the text), conform essentially to Letter Symbols for Structural Analysis (ASA–Z10.8–1949) prepared by the American Standards Association, with ASCE participation, and approved by the Association in 1949. For

Note.—Written comments are invited for publication; the last discussion should be submitted by March 1, 1951.

[1] Cons. Engr., Houston, Tex.

reinforced footings, seven variables are involved in the three sets of curves (Figs. 1, 2, and 3)—namely, $\frac{P}{(f_c)'}$, $\frac{d}{B}$, q, Σo, y, $\frac{v_c}{(f_c)'}$, and $f_s \frac{A_s}{B\,P}$. Their significance, where not conforming to the standard, is made clear by Fig. 4.

Shear (Fig. 1).—The critical section for shear is defined by the code as a section parallel to, and at a distance d from, the face of the column, d being

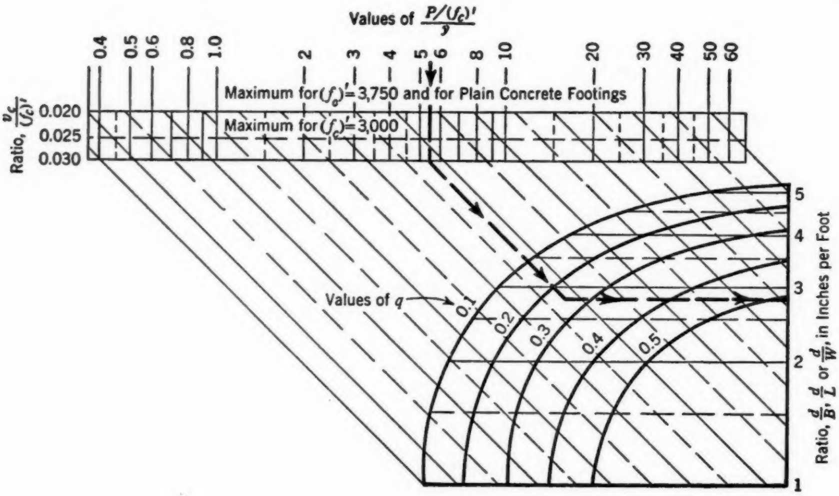

FIG. 1.—VALUES OF $\frac{P/(f_c)'}{y}$, $\frac{v_c}{(f_c)'}$, AND $\frac{d}{B}$

the effective depth of the footing at the base of the column, in inches. The external shear carried by this section is equal to the load on an area bounded by the section, 45° diagonals from the corners of the column, and the included edges of the footing. From Fig. 4, the length (l) of the section is

$$l = q\,b + 2\,d = b\left(q + \frac{2\,d}{b}\right) \qquad (1)$$

in which b is the width of a square footing, in inches, and q is the ratio of the width of the column to the width of the footing. The external shear carried is

$$V = \frac{P\,b^2\left[1 - \left(q + \frac{2\,d}{b}\right)^2\right]}{144 \times 4} \qquad (2)$$

in which V is the total shear on the section, in pounds per inch of width, and P is the load on the footing (soil bearing minus the weight of the footing), in pounds per square foot.

Let v_c be the unit shear on the section, in pounds per square inch; y be the ratio of the effective depth at the shear section to the effective depth at the column; and $(f_c)'$ be the compressive strength of concrete at twenty-eight days, in pounds per square inch. The standard concrete symbol j is the ratio of the

distance between the resultant compressive stress and the resultant tensile stress, to the distance from the outer compressive fiber to the resultant tensile stress. Then,

$$v_c = \frac{P\,b^2\left[1 - \left(q + \frac{2\,d}{b}\right)^2\right]}{576\,b\left(q + \frac{2\,d}{b}\right) j\,y\,d} \qquad (3)$$

and

$$\frac{v_c}{(f_c)'} = \frac{0.002\left[1 - \left(q + \frac{2\,d}{b}\right)^2\right]}{\left(q + \frac{2\,d}{b}\right)\frac{d}{b}}\left[\frac{\frac{P}{(f_c)'}}{y}\right] \qquad (4a)$$

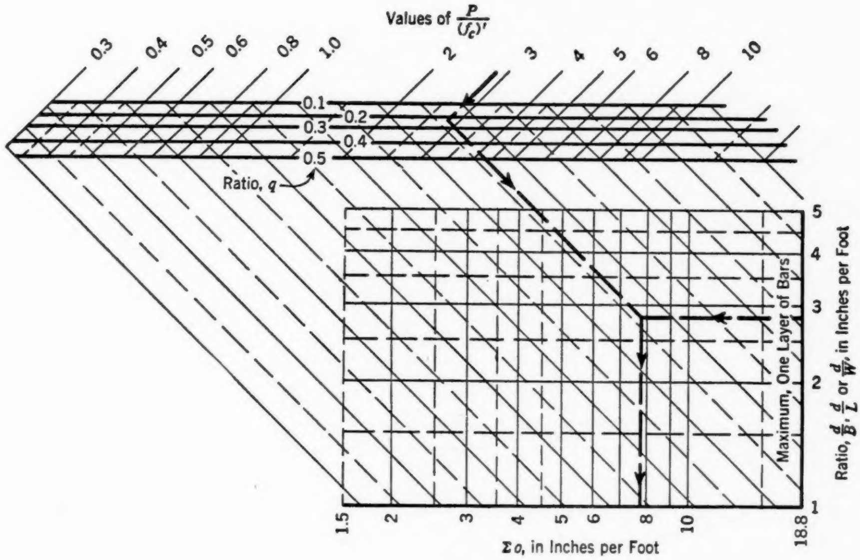

FIG. 2.—VALUES OF $\frac{P}{(f_c)'}$, $\frac{d}{B}$, AND Σo

If B $(= b/12)$ is the width of a square footing, in feet, Eq. 4a becomes

$$\frac{v_c}{(f_c)'} = \frac{0.004\left[36 - \left(6\,q + \frac{d}{B}\right)^2\right]}{\left(6\,q + \frac{d}{B}\right)\frac{d}{B}}\left[\frac{\frac{P}{(f_c)'}}{y}\right] \qquad (4b)$$

Bond (Fig. 2).—The critical section for bond is specified as the same as that for bending moment and is at the face of the column. Its length is the full width of the footing. The code gives the permissible bond stress in a two-way footing as 0.056 $(f_c)'$, with a maximum of 200 lb per sq in. It also specifies that not less than 85% of the external shear carried by the section shall be

considered in computing bond. Since 85% of 0.056 $(f_c)'$ for 3,750-lb concrete is only 210 lb per sq in., and since no reduction in the external shear need be made,[2] the 85% allowance and the 200-lb maximum have been ignored in the equation for bond. Thus, the external shear per inch of width is

$$V = \frac{P\,b\,(1 - q)}{144 \times 2} \qquad (5)$$

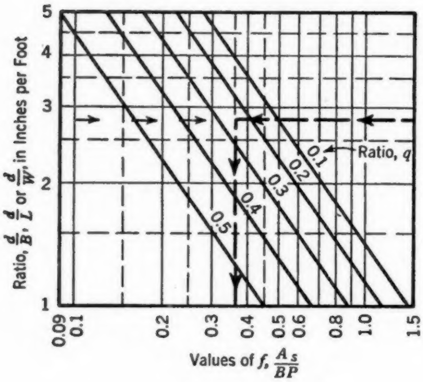

FIG. 3.—VALUES OF $\frac{d}{B}$ AND $f_s \frac{A_s}{B\,P}$

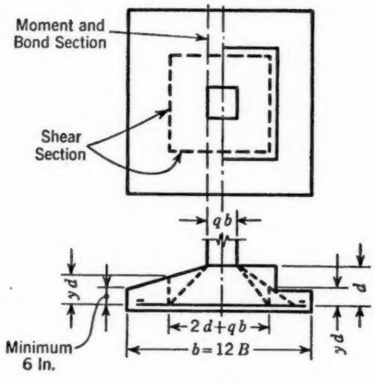

FIG. 4

The sum of the perimeters of the reinforcing bars, in inches per inch in width of footing, is

$$\Sigma o = \frac{V}{u\,j\,d} = \frac{0.071\,\dfrac{P\,(1 - q)}{(f_c)'}}{\dfrac{d}{b}} \qquad (6)$$

in which u is the allowable bond stress. To convert to inches per foot in width of footing, substitute B for b in Eq. 6.

Bending Moment (*Fig. 3*).—The moment M per inch in width is

$$M = \frac{P\,b^2\,(1 - q)^2}{144 \times 8} = f_s\,A_s\,j\,d \qquad (7)$$

in which f_s is the tensile stress in the reinforcing steel and A_s is the section area of steel per inch width of footing. Using an average value of $j = 0.833$,

$$\frac{f_s\,A_s}{b\,P} = 0.00104\,\frac{(1 - q)^2}{\dfrac{d}{b}} \qquad (8a)$$

If A_s is the steel area per foot, and $b = 12\,B$,

$$\frac{f_s\,A_s}{B\,P} = 1.80\,\frac{(1 - q)^2}{\dfrac{d}{B}} \qquad (8b)$$

[2] "Reinforced Concrete Wall and Column Footings," by Frank E. Richart, *Proceedings*, A.C.I., Vol. 45, 1948–1949, p. 97, title 45–6a, and p. 237, title 45–6b.

Since $f_s A_s$ varies inversely with j, the tension $f_s A_s$ for other values of j may be found by multiplying the values from Fig. 3 by $0.833/j$.

As in the case of bond, considering recent tests,[2] the 85% allowance has been ignored in Eqs. 8.

Plain Concrete Footings (Fig. 5).—The permissible tension in the concrete is limited to $0.03\ (f_c)'$ on a section at the face of the column. The moment per inch of width is

$$M = \frac{P\, b^2 (1 - q)^2}{144 \times 8} = f_c\, Z_c = \frac{0.03\ (f_c)'\ d^2}{6} \qquad (9)$$

in which f_c is the allowable compressive stress in concrete, in pounds per square inch, and Z_c is the section modulus of a plain concrete footing. Solving for d/b,

$$\frac{d}{b} = 0.417\ (1 - q) \left[\frac{P}{(f_c)'} \right]^{0.5} \qquad (10a)$$

With the width expressed in feet,

$$\frac{d}{B} = 4.99\ (1 - q) \left[\frac{P}{(f_c)'} \right]^{0.5} \qquad (10b)$$

Explanation of Curves and Examples of Their Use

Fig. 1 is a set of curves that contains the variables $\frac{P/(f_c)'}{y}$, q, $v_c/(f_c)'$, and d/B, and gives the value of d/B when the other three are known. Fig. 2 contains the variables $P/(f_c)'$, q, d/B, and Σo, and gives the value of Σo when the other three are known. Fig. 3 contains the variables d/B, q, and $\frac{f_s A_s}{B\, P}$, and gives the value of the latter when the others are known. For plain concrete footings, Fig. 5 is a set of curves based on tension in the concrete, containing the variables d/B, q, and $P/(f_c)'$, which gives the value of d/B when the other two are known. In designing footings, the total superimposed load, the allowable soil bearing pressure, the column size ($q\ b$), and the concrete strength $(f_c)'$ are specified, or known. The width of the footing (B) is determined by the superimposed load and the allowable bearing; therefore, the problem is to determine the most satisfactory proportions for d/B, y, and A_s to fit the requirements. As the design procedure

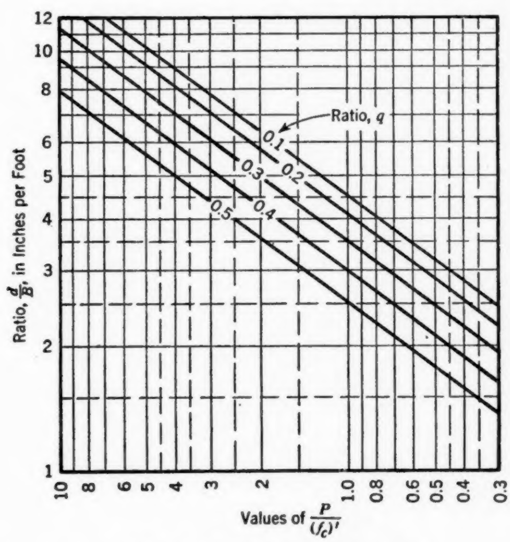

Fig. 5.—Plain Concrete Footings

for all types of footings on soil is essentially the same, only one example of a square footing and one of a rectangular footing will be given, together with a brief statement of the modifications necessary to adapt the curves to the design of rectangular, one-way, and continuous footings.

Design of Square Footings; Numerical Examples

Example 1, Square Footing.—Assume a 24-in. by 24-in. square column (area = 4 sq ft), carrying a load of 360 kips. The allowable soil bearing pressure is 6,000 lb per sq ft and the specified compressive strength of concrete $(f_c)'$ at twenty-eight days is 2,000 lb per sq in. The estimated weight of the footing being 300 lb per sq ft, $P = 6{,}000 - 300 = 5{,}700$ lb per sq ft. The area required to support 360 kips, therefore, is $360/5.7 = 62.1$ sq ft—say, an 8-ft square footing with an area of 64 sq ft. A corrected net load is $360/64 = 5.6$ kips per sq ft, from which $P/(f_c)' = 5{,}600/2{,}000 = 2.8$. The width ratio q is $2/8 = 0.25$ and (assuming $y = 0.50$), $P/(f_c)' = 2.8/0.5 = 5.6$.

Enter Fig. 1 at $\dfrac{P/(f_c)'}{y} = 5.6$; follow vertically to $v_c/(f_c)' = 0.03$; thence down along diagonal to $q = 0.25$; from there horizontally to find $d/B = 2.8$ in. per ft. Enter Fig. 2 with $P/(f_c)' = 2.8$; follow diagonal down to left to $q = 0.25$; then down diagonal to right to an intersection with the horizontal $d/B = 2.8$; thence vertically to find $\Sigma o = 7.8$ in. per ft. The use of $\frac{5}{8}$-in. bars at 3-in. centers gives $\Sigma o = 7.8$ in. and $A_s = 1.24$ sq in. per foot width of footing. In Fig. 3, with $d/B = 2.8$ and $q = 0.25$, $\dfrac{f_s\,A_s}{B\,P} = 0.36$, from which $f_s = \dfrac{0.36 \times 8 \times 5{,}600}{1.24} = 13{,}000$ lb per sq in.

Example 2, Rectangular Footing.—With the same basic data as in Example 1, assume a maximum footing width W of 7 ft. The area required being 64 sq ft, the length L of the footing is $64/7 = 9.14$ ft. Then $q_L = 2/9.14 = 0.22$; and $q_W = 2/7 = 0.29$. As in Example 1, $P = 5{,}600$ lb per sq ft and $P/(f_c)' = 2.8$. Assume $y = 0.5$, then $\dfrac{P/(f_c)'}{y} = 5.6$.

With the latter value (5.6) enter Fig. 2 and move vertically to $v_c/(f_c)' = 0.03$; thence diagonally downward and to the right, intersecting $q_L = 0.22$ at $d/L = 2.9$ in. per foot length of footing. If $d/L = 2.9$, $d = 2.9 \times 9.14 = 26.5$ in. As a check, enter Fig. 2 as before but intersecting $q_W = 0.29$ at $d/W = 2.7$ in. per foot width of footing. If $d/W = 2.7$, $d = 2.7 \times 7 = 18.9$ in. The greater of the two depths (26.5 in.) is the one selected.

From Fig. 2, $\Sigma o_L = 7.6$ in. per ft, requiring $\frac{5}{8}$-in. bars on 3-in. centers—that is, $A_{sL} = 1.24$ sq in. per ft and (from Fig. 3), $\dfrac{f_s\,A_s}{L\,P} = 0.38$. Consequently (from Fig. 3), $f_{sL} = \dfrac{0.38 \times 9.14 \times 5{,}600}{1.24} = 15{,}700$ lb per sq in.

As a check, with $d = 26.5$ in., $d/W = 3.8$ in. per ft. In Fig. 2, begin with $q_W = 0.29$ at $d/W = 3.8$; then Σo_W (from Fig. 2) $= 5.8$ in. per ft. Using $\frac{1}{2}$-in. bars, 3 in. on centers, $\Sigma o_W = 6.3$ in.; that is, $A_{sW} = 0.80$ sq in. per ft,

and (from Fig. 3), $\frac{f_s A_s}{W P} = 0.24$. As before, $f_s W = \frac{0.24 \times 7 \times 5{,}600}{0.80} = 11{,}800$ lb per sq in.

Example 3, One-Way and Continuous Footings.—Since one-way and continuous footings are normally of uniform thickness, they will be controlled by bond. The ACI code increases the bond from 0.056 $(f_c)'$ in two-way footings to 0.075 $(f_c)'$ in one-way footings. Therefore, for one-way footings, decrease $P/(f_c)'$ by 25% (in Fig. 3 only) and proceed as for the two-way footing.

Summary

The curves are more flexible and cover a wider range of design conditions than the most comprehensive tables. Their use entails a minimum of calculation and shows clearly the design procedure and the relation between the design conditions.

The resultant design will be within the practical limits of loading and soil bearing determination.